NORMAL STRESS DUE TO NORMAL FORCE

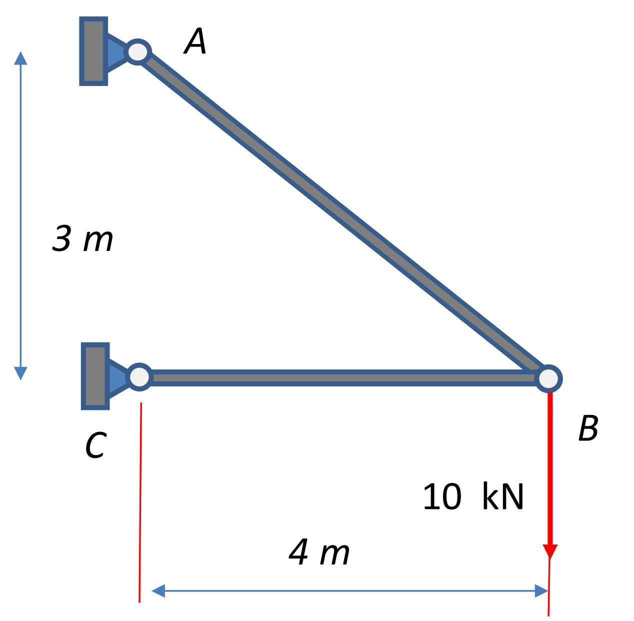
I) The bars AB and BC are articulated on their extremes and withstand a vertical load of 10 kN at point B.

The material of the structure is a certain type of steel whose yield stress is equal to 240 MPa.

a) DSI of the structure

b) Determine the minimum radius for each member to be able to withstand such condition of load with a safety coefficient C_s = 4. The stretches are circular.

c) What would be the length of the side of another squared cross section, to withstand the same forces? Would it be greater for the radius or the square?



a) DSI

$$DSI = r + m - 2j = 4 + 2 - 2.3 = 0$$

b) Minimum Radius

Note that in this exercise it is not necessary to calculate the reaction forces. Isolating nodes

$$\sum F_x = 0 \rightarrow -\frac{4}{5}F_{AB} + F_{BC} = 0$$

$$\sum F_x = 0 \rightarrow -\frac{3}{5}F_{AB} - 10 = 0$$

Therefore:

$$F_{AB} = \frac{50}{3}\text{kN};\ F_{BC} = \frac{40}{3}\text{kN}$$

$$SF = 2 = \frac{\sigma_{max}}{\sigma_{serv}} = \frac{240}{4} \rightarrow \sigma_{serv} = 60 \text{ MPa}$$

Now analyzing the cross section:

$$A_{ab} = \frac{F_{ab}}{\sigma_{serv}} = 69{,}45 \text{ mm}^2$$

$$A_{bc} = \frac{F_{bc}}{\sigma_{serv}} = 55{,}56 \text{ mm}^2$$

$$A_{ab} = \frac{\pi . r_{ab}^2}{4} \rightarrow \mathbf{r_{ab} = 9{,}40\ mm}$$

$$A_{bc} = \frac{\pi . r_{bc}^2}{4} \rightarrow \mathbf{r_{bc} = 8{,}41\ mm}$$

As it is seen AB stretch is the limiting stretch.

c) Size of the square.

Now, with the square cross section of side a:

$$\frac{\pi . r_{ab}^2}{4} = a^2 \rightarrow a = 8{,}33 < \text{Radius}$$

II) A column of section axb=100x50 mm; l=2 m Δl=1 mm; Δb=-0,007 mm; Tensile force of F=50 tm exerted in its cross section. Calculate:

a) Longitudinal Young modulus

b) Poisson coefficient

c) Increment of length of a

$$\mathbf{E} = \frac{\sigma}{\varepsilon} = \frac{Fl}{ab\Delta l} = \frac{50.10^3.2.10^2}{10.5.0{,}1} = \mathbf{2.10^6\ kp/cm^2}$$

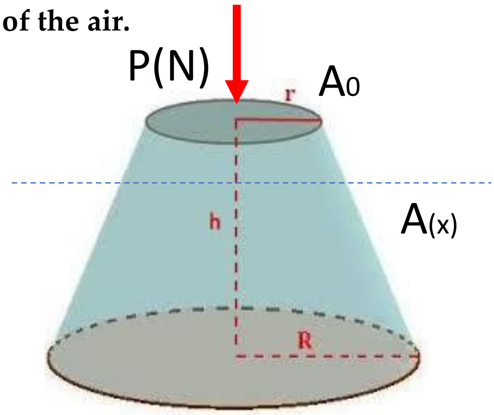
$$\frac{\Delta b}{b} = -\mu\varepsilon = -\frac{\mu\Delta l}{l}$$

$$\boldsymbol{\mu} = -\frac{l.\Delta b}{b.\Delta l} = \frac{2000}{50}.\frac{0{,}007}{1} = \mathbf{0{,}28}$$

$$\frac{\Delta a}{a} = -\mu\varepsilon = -\frac{\mu\Delta l}{l}$$

$$\mathbf{\Delta a} = -\frac{\mu\Delta l}{l}a = -0{,}28.\frac{100}{2000} = \mathbf{-0{,}014\ mm}$$

III) Find the area in function of x of a generic body subjected to a force P and its own weight (with linear density γ) in order to keep constant the stress along the body. Compare these results with the variation of pressure in function of density of the air.



Isolating a differential of the element we will analyze the effect of the different forces

$$dN = \sigma . dA \rightarrow \frac{P}{A_0} dA = \gamma . A . dx$$

$$\int \frac{dA}{A} = \int \frac{\gamma . A_0}{P} dx$$

$$\ln A = \ln C + \frac{\gamma . A_0}{P} x \rightarrow A(x) = Ce^{\frac{\gamma . A_0}{P} x}$$

Boundary conditions:

$$A(x = 0) = A_0 \rightarrow \mathbf{A(x) = A_0 e^{\frac{\gamma . A_0}{P} x}}$$

<u>Variation of pressure with the air</u>:

$$P + dP = P + \rho g dz \rightarrow dP = \rho g dz$$

$$PV = nRT = \frac{m}{M} RT \rightarrow \rho = \frac{PM}{RT}$$

$$dP = \frac{PM}{RT} g dz \rightarrow \frac{dP}{P} = \frac{Mg}{RT} dz$$

$$\int \frac{dP}{P} = \int \frac{Mg}{RT} dz$$

$$\ln P = \ln C + \frac{Mg}{RT} z \rightarrow P(z) = Ce^{\frac{Mg}{RT} z}$$

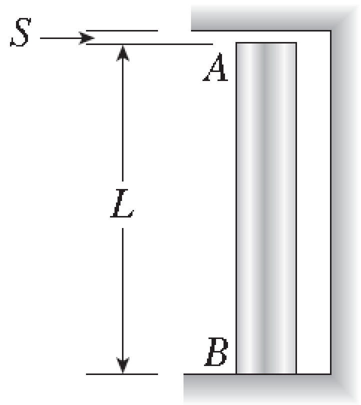
$$P(z = 0) = P_0 \rightarrow \mathbf{P(z) = P_0 e^{\frac{Mg}{RT} z}}$$

Both phenomena respond to an analogue equation because the air can be considered as a fluid.

IV) A solid of cylindrical shape of 250 mm of height is placed clamped vertically but leaving a small distance s of 0,08 mm. Determine the stress that the body will experiment due to an increment of temperature of 50°C with and without the small tolerance s.

Data:

E = 200 GPa $\boldsymbol{\alpha = 9,6.10^{-6\circ} C^{-1}}$



In order to calculate the stress due to the thermal effect the next expression is used:

$$\sigma = E\varepsilon = E \frac{\Delta l}{l} = \frac{E}{l} (\alpha l \Delta t - s)$$

When considering tolerances:

$$\sigma = \frac{200 \text{ GPa}}{250 \text{ mm}} (9{,}6.10^{-6\circ} C^{-1}. 250\text{mm}. 50°C - 0{,}08 \text{ mm})$$

$$\boldsymbol{\sigma = 32 \text{ MPa}}$$

Without tolerances:

$$\sigma = \frac{200 \text{ GPa}}{250 \text{ mm}} (9{,}6.10^{-6\circ} C^{-1}. 250\text{mm}. 50°C - 0 \text{ mm})$$

$$\boldsymbol{\sigma = 96 \text{ MPa}}$$

<u>Thus, leaving a small space of 0,08 mm it is possible to reduce the stress on the body roughly three times</u>. This is showing the importance of tolerances in the design of structures, systems and components.